A SIMPLE MODEL OF AN ICE SHEET "TIPPING POINT": THE HEIGHT-MASS BALANCE FEEDBACK

In the first lecture, we derived a simple model of glacier length L,

$$L = 2(H + B_0 - E)/\beta,$$

where H is the average glacier thickness, β is the bed slope, B_0 is the bed elevation at the glacier toe, and E is the ELA.

Consider the mean thickness H averaged over the entire glacier length. In the first lecture we treated H as being independent of L but we can do better than that. Oerlemans proposes

$$H = \frac{\alpha\sqrt{L}}{1 + \nu\beta}$$

Combining these two equations gives,

$$L = 2\left(\frac{\alpha\sqrt{L}}{1 + \nu\beta} + B_0 - E\right)/\beta.$$

Defining $N = \sqrt{L}$,

$$N^2 = 2\left(\frac{\alpha N}{1 + \nu\beta} - R\right)/\beta,$$

which is a quadratic equation and we defined $R = E - B_0$. The solution is

$$L = \frac{1}{2}\left[\frac{2\alpha}{\beta(1 + \nu\beta)} \pm \sqrt{D}\right]^2$$

where D is the discriminant. Although there are two solutions, only the positive sign is stable. As a quadratic equation, real-valued solutions exist when the discriminant is positive. The discriminant is

$$D = \frac{4\alpha^2}{\beta^2(1 + \nu\beta)^2} - \frac{8R}{\beta}$$

The first term is always positive, which suggests that only certain values of R have solutions. Apparently glaciers disappear for at a critical value of the ELA. This occurs when $D = 0$. Solving for this value, we find,

$$E_{crit} = \frac{\alpha^2}{2\beta(1 + \nu\beta)^2} + B_0$$

So if a glacier initially has some size $L > 0$ and then the ELA moves up due to warming, the glacier will disappear when $E = E_{crit} > 0$. The length at this critical point is,

$$L_{crit} = \frac{\alpha}{\beta(1 + \nu\beta)} \approx 5.5 \text{ km}$$

where we have used values proposed by Oerlemans for a mountain glacier: $\alpha = 0.5$, $\beta = 0.03$, $\nu = 10$.