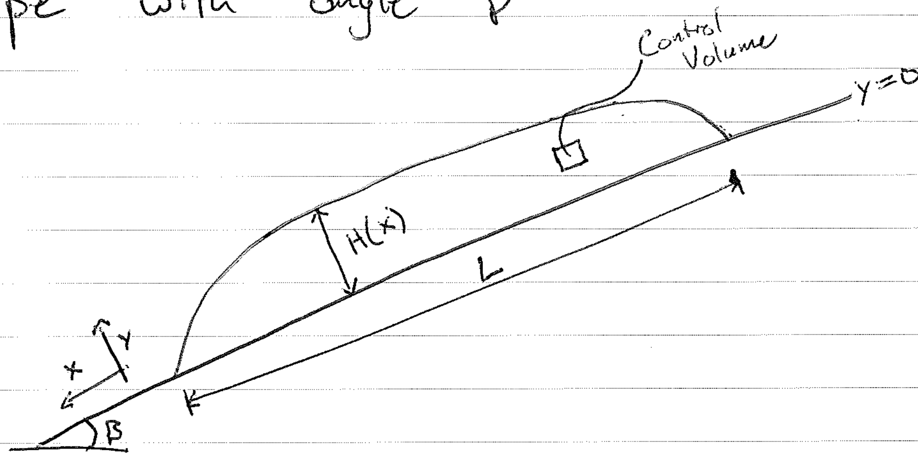
The Simplest Glacier Model

Consider a glacier of length L resting on a slope with angle β



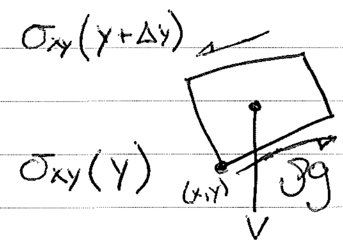
The total mass is $M = \int \rho \, d^3x$, volume $V = \int d^3x$.

Volume changes occur from accumulation (snow fall, avalanching, wind transport) and ablation (runoff, sublimation, etc),

$$\frac{dV}{dt} = \underbrace{\int_{\substack{\text{glacier}\\ \text{surface}}} \overbrace{\dot{m}}^{\text{specific mass balance rate (m. ice equiv/yr)}}(\vec{x}, t) \, d^2x}_{\text{"Sources of mass"}} \qquad (1)$$

This is a statement of <u>Mass Conservation</u>.

We next consider the <u>conservation of momentum</u> within a small control volume. A free body diagram looks like this:



(We work in stresses, with SI unit $1 \text{ Pa} \equiv 1 \text{ N/m}^2$)

We consider momentum only in the down-glacier x direction. <u>Force Balance</u> dictates that,

$$\sigma_{xy}(y+\Delta y)\,\Delta x \Delta z - \sigma_{xy}(y)\,\Delta x \Delta z \quad + \rho g \sin\beta \,\Delta x \Delta y \Delta z = 0$$

In the limit of a small control volume, we have

$$\frac{\partial \sigma_{xy}}{\partial y} = -\rho g \sin \beta \qquad \underline{\text{Conservation of momentum}} \qquad (2)$$

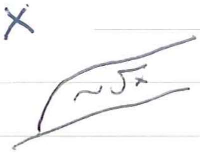
Integration gives

$$\overbrace{\sigma_{xy}(y = h(x))}^{\text{Zero at free surface}} - \underbrace{\sigma_{xy}(y=0)}_{\text{Bed shear stress}} = -\rho g H \sin\beta$$

In our simple model, $\beta \approx$ surface slope so that for small slopes, $\sigma_{xy}(y=0) = \rho g H \beta \approx \rho g H \frac{\partial H}{\partial x}$

We assume, following very good agreement with data, that the bed shear stress takes on an approximately constant value, $\sigma_{xy}(0) = \tau_0$. Then,

$$\rho g H \frac{\partial H}{\partial x} = \frac{1}{2}\rho g \frac{\partial (H^2)}{\partial x} = \tau_0 \Rightarrow H(x) = \sqrt{\frac{2\tau_0}{\rho g}\, x}$$



This is the "parabolic ice sheet model"

The average thickness is $\bar{H} = \frac{1}{L}\int_0^L H(x)\,dx$
$= \sqrt{2L\tau_0/\rho g}$

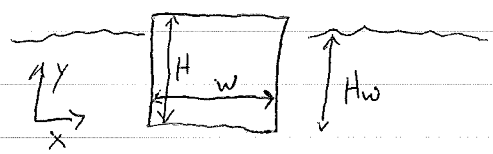
A warmup exercise

Question: How much of an ice berg is below water, assuming that the ice berg is freely floating.

If you know how to do this, consider writing an answer starting with

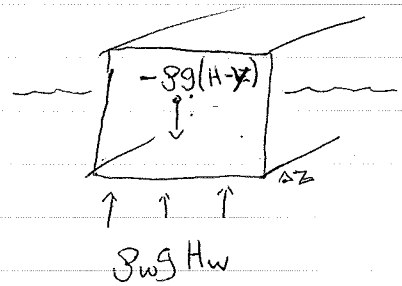
$$\frac{dP}{dt} = \int_S \text{Applied Loads } dS + \int_{\text{Volume}} \text{Body forces } dV$$

If not, start with a free body diagram,



And remember that the vertical (y) momentum balance depends on forces acting in the vertical direction.

Answer



Force Balance:

$$-\rho_i g H W \Delta Z + \rho_w g H_w W \Delta Z = 0$$

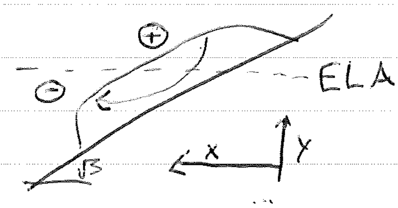
$$\Rightarrow \frac{H_w}{H} = \frac{\rho_i}{\rho_g}$$

(Obviously W and ΔZ cancel, but they allowed us to do a force- rather than a stress balance.)

Application to LGM Morraines

Suppose mass balance rates follow

$$\dot{m} = \gamma \, (h - ELA)$$



Now consider <u>elevation</u> based coordinates.

Surface elevation is $h = H(x) + B_0 + \beta x$

At equilibrium,

$$\int_0^L \dot{m}\, dx = 0 \Rightarrow \underbrace{\int H(x)\, dx}_{\equiv L\bar{H}} + (B_0 - E)L + \tfrac{1}{2}\beta L^2 = 0$$

$$\Rightarrow L = \frac{-2}{\beta}\left(B_0 - E + \bar{H}\right)$$

$$= \frac{-2}{\beta}\left(B_0 - E + \overbrace{\tau_0/\rho g \beta}^{\text{linear model}}\right)$$

So then $\quad \dfrac{\Delta L}{\Delta E} \approx \dfrac{dL}{dE} = \dfrac{2}{-\beta}$

Consider the Middle Teton Glacier (photo in slides).

$\Delta L \approx -5\,\text{Km}$, $\beta \approx \frac{1\,\text{Km}}{5\,\text{Km}} = 0.2$,

$\Delta(ELA) \approx 500\,m$

Global mean LGM $\Delta ELA \sim 1\,\text{Km}$