Glaciology Problem Set 2

Due in class on Friday October 26, 2018.

1 Basal shear stress due to regelation and viscous flow

Basal shear stress occurs in this model due to the component of pressure that acts against bed topography,

$$\tau = \frac{1}{A}\int_A \left(P\frac{\partial B}{\partial x}\right) dA \tag{1}$$

Here, A is the bed area of the glacier.

1.1 Plancherel's theorem

We first invoke Plancherel's theorem,

$$\int_A f(\mathbf{x})g(\mathbf{x})dA = \int_A \hat{f}(\mathbf{k})\hat{g}(\mathbf{k})d\mathbf{k}$$

Apply this relationship to Equation 1.

1.2 The roughness parameter, ζ

At this point, it is convenient to denote the *nondimensional roughness* as,

$$\hat{\zeta}^2 \equiv A\left(\ell^2\hat{B}\right)^2$$

1.3 Isotropic roughness

We can simplify the analysis by assuming that the roughness is just a function of $\ell = \sqrt{k^2+h^2}$. Carry out a change of variables (analogous to polar coordinates) on the above integration and evaluate the resulting integrals.

1.4 Uniform roughness spectrum

Assume that the roughness spectrum is constant across all wavelengths, i.e., $\zeta(\ell) = \zeta_0$. Evaluating all integrals results in a smiple relationship between bed shear stress and sliding velocity.

2 Ice shelf velocities revisited: accounting for strain thinning

In class we wrote out an expression for the ice shelf strain rate ϵ_{xx} as a function of the ice thickness. In the simple geometry we considered in class we can actually analytically account for a nonlinear ice rheology. We define the strain rate dependent viscosity to be,

$$\eta = B\epsilon_E^{1/n-1},$$

where a typically $n = 3$ and $B = 1$ MPa s$^{1/3}$. The effective strain rate is

$$\epsilon_E \equiv \sqrt{\frac{\epsilon_{xx}^2 + \epsilon_{zz}^2}{2}}.$$

Use mass balance to simplify the effective stress and then solve the resulting integral for the ice velocity $u(x)$.

3 Grounding line motion with GIA

The weight of ice sheets cause the surface of the Earth to deform. This is commonly called glacial isostatic adjustment or GIA. If the surface of the Earth were initially at $z = 0$, the deflection due to this isostatic adjustment results in an ice sheet with a basal topography $z = B(x)$ that is proportional to the ice thickness h according to

$$B(x) = -\frac{\rho_i}{\rho_m}h(x)$$

where ρ_i/ρ_m is the ratio between the density of ice and the density of the Earth's crust and upper mantle, typically on the order of 1/3.

Generalize the treatment of the marine ice sheet grounding line equation from class to incorporate an isostatically adjusted bed. What effect does GIA have on grounding line position?